Engg 6410

Suggested Problems 1

Formulation Problems

1. A company that operates 10 hours a day manufactures two products on three sequential processes. The following table summarizes the data of the problem:

Minutes per unit

Product	Process 1	Process 2	Process 3	Unit profit
1	10	6	8	\$2
2	5	20	10	\$3

Formulate the problem to find the optimal mix of the two products.

Solution

Let x1 be the quantity of product 1 produced

Let x2 be the quantity of product 2 produced

Max Z = 2x1 + 3x2

s.t.

10x1 + 5x2 ≤ 10*60

6x1 + 20x2 ≤ 10*60

8x1 + 10x2 ≤ 10*60

x1, x2 ≥ 0

2. A company produces two products, A and B. The sales volume for A is at least 80% of the total sales of both A and B. However, the company cannot sell more than 100 units of A per day. Both products use one raw material, of which the maximum daily availability is 240 lb. The usage rates of the raw material are 2 lb per unit of A and 4 lb per unit of B. The profit units for A and B are $20 and $50 respectively. Formulate the model to determine the optimal product mix for the company.

   Solution

   Let x1 be quantity of product A produced
   Let x2 be quantity of product B produced

   Max Z = 20x1 + 50x2
   s.t.
   $x1 \leq 100$
   $x1 \geq 0.8 * (x1 + x2) \Rightarrow 0.2x1 - 0.8x2 \geq 0$
   $2x1 + 4x2 \leq 240$
   $x1, x2 \geq 0$

3. The Continuing Education Division at the Ozark Community College offers a total of 30 courses each semester. The courses offered are usually of two types: practical and humanisitic. To satisfy the demands of the community, at least 10 courses of each type must be offered each semester. The division estimates that the revenues of offering practical and humanistic courses are approximately $1500 and $1000 per course, respectively. Formulate a model that will provide the optimal course offering for the college.

Solution

Let x1 be number of practical courses offered
Let x2 be number of humanistic courses offered

Max Z = 1500x1 + 1000 x2
s.t.
x1 + x2 = 30
$x1 \geq 10$
$x2 \geq 10$
x1, x2 integer

4. In the Ma-and-Pa grocery store, shelf space is limited and must be used effectively to increase profit. Two cereal items, Grano and Wheatie, compete for a total shelf space of 60 ft^2. A box of Grano occupies 0.2 ft^2 and a box of Wheatie needs 0.4 ft^2. The maximum daily demands of Grano and Wheatie are 200 and 120 boxes, respectively. A box of Grano nets $1.00 in profit and a box of Wheatie $1.35. Ma-and-Pa thinks that because the unit profit of Wheatie is 35% higher than that of Grano, Wheatie should be allocated 35% more space than Grano, which amounts to allocating 57% to Wheatie and 43% to Grano. Formulate the model that would allow you to determine if this is true.

Solution

Let x1 be number of boxes of Grano
Let x2 be number of boxes of Wheatie

Max Z = x1 + 1.35x2
s.t.
0.2x1 + 0.4x2 ≤ 60
x1 ≤ 200
x2 ≤ 120
x1, x2 ≥ 0

When solved, this problem will give a solution that meets the required allocation.

5. Show & Sell can advertise its products on local radio and television (TV). The advertising budget is limited to $10,000 a month. Each minute of radio advertising costs $15 and each minute of TV commercials is $300. Show & Sell likes to advertise on radio at least twice as much as on TV. In the meantime, it is not practical to use more than 400 minutes of radio advertising a month. From past experience, advertising on TV is estimated to be 25 times as effective as on radio. Formulate the model that will allow you to determine the optimum allocation of the budget to radio and TV advertising.

Solution

Let x1 be # minutes allocated to radio
Let x2 be # minutes allocated to tv

Max Z = x1 + 25x2
s.t.
15x1 + 300x2 ≤ 10000
x1 ≥ 2x2 => x1 – 2x2 ≥ 0
x1 ≤ 400
x1, x2 ≥ 0

6. Toolco has contracted with AutoMate to supply their automotive discount stores with wrenches and chisels. AutoMate's weekly demand consists of at least 1500 wrenches and 1200 chisels. Toolco cannot produce all the requested units with its present one-shift capacity and must use overtime and possibly subcontract with other tool shops. The result is an increase in the production cost per unit, as shown in the following table. Market demand restricts the ratio of chisels to wrenches to at least 2:1.

Tool	Production type	Weekly production range (units)	Unit cost ($)
Wrenches	Regular	0 – 550	2.00
	Overtime	551 – 800	2.80
	Subcontracting	801 - ∞	3.00
Chisel	Regular	0 – 620	2.10
	Overtime	621 – 900	3.20
	Subcontracting	901 - ∞	4.20

Formulate the problem as a linear program.

Solution

Let x11 be # of chisels produced in regular time
Let x12 be # of chisels produced in overtime
Let x13 be # of chisels produced using subcontracting
Let x21 be # of wrenches produced in regular time
Let x22 be # of wrenches produced in overtime
Let x23 be # of wrenches produced using subcontracting

Min Z = 2.1 x11 + 3.2 x12 + 4.2 x13 + 2 x21 + 2.8 x22 + 3 x23
s.t.
$x21 + x22 + x23 \geq 1500$
$x11 + x12 + x13 \geq 1200$
$2(x21 + x22 + x23) \leq (x11 + x12 + x13)$ => $(x11 + x12 + x13) - 2(x21 + x22 + x23) \geq 0$
$x21 \leq 550$
$x22 \leq 800 – 550$
$x11 \leq 620$
$x12 \leq 900 – 620$
$x11, x12, x13, x21, x22, x23 \geq 0$