Engg 6140

Suggested Problems 2

1. A company that operates 10 hours a day manufactures two products on three sequential processes. The following table summarizes the data of the problem:

| Minutes per unit | | | | |
|---|---|---|---|---|
| Product | Process 1 | Process 2 | Process 3 | Unit profit |
| 1 | 10 | 6 | 8 | $2 |
| 2 | 5 | 20 | 10 | $3 |

Determine the optimal mix of the two products.

***Solution***

Let $x_1$ be the quantity of product 1 to produce

Let $x_2$ be the quantity of product 2 to produce

Max $Z = 2x_1 + 3x_2$

s.t.

$10x_1 + 5x_2 \leq 10 * 60$ minutes per day

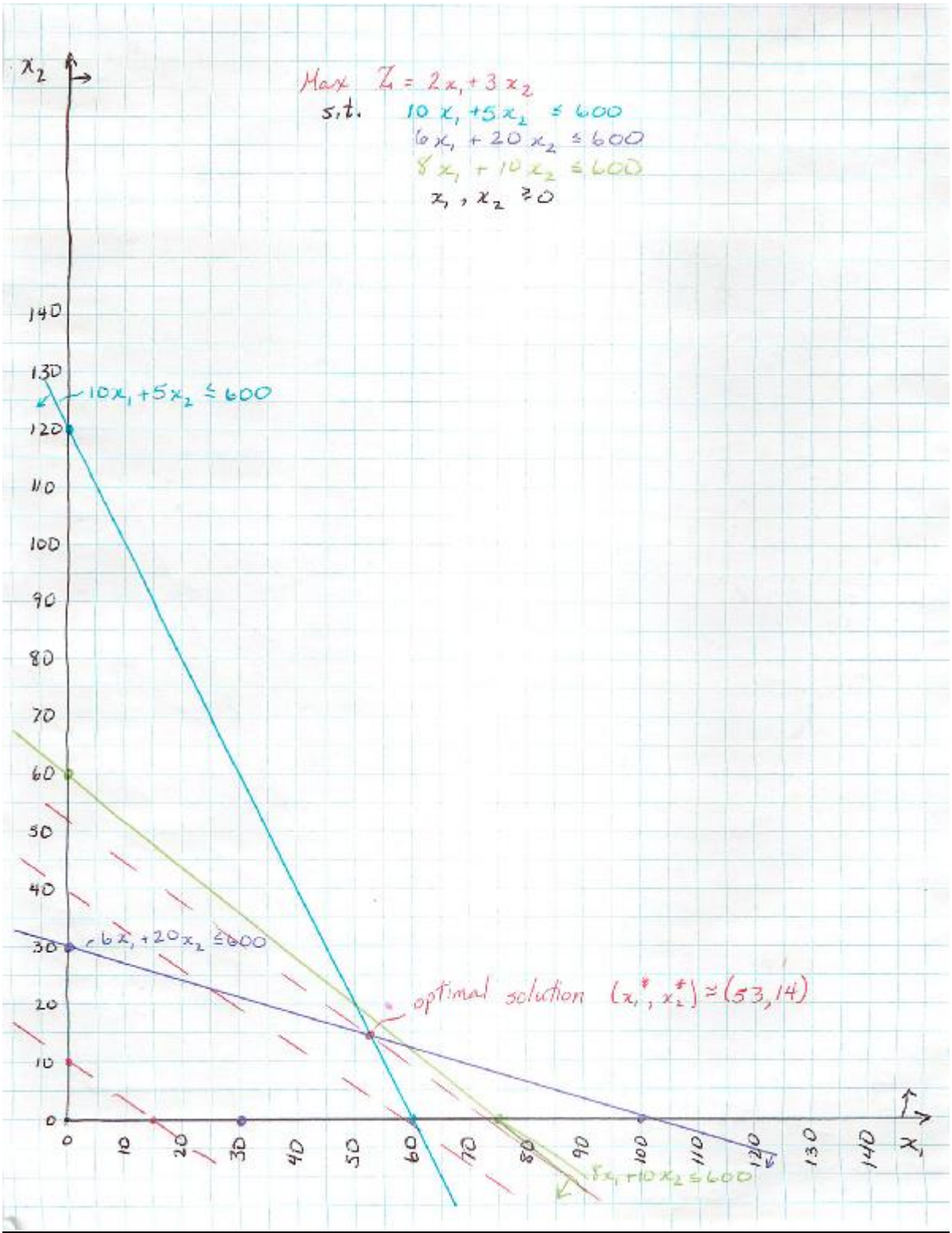
$6x_1 + 20x_2 \leq 10 * 60$ minutes per day

$8x_1 + 10x_2 \leq 10 * 60$ minutes per day

$X_1, x_2 \geq 0$

Since this problem has only two decision variables, it can be solved graphically or using Simplex.

*Graphical Solution*




*Simplex Solution*

See Excel file

2. Jack is an aspiring freshman at Ulern University. He realizes that "all work and no play make Jack a dull boy." Jack wants to apportion his available time of about 10 hours a day between work and play. He estimates that play is twice as much fun as work. He also wants to study at least as much as he plays. However, Jack realizes that if he is going to get all his homework assignments done, he cannot play more than 4 hours a day. How should Jack allocate his time to maximize his pleasure from both work and play?

***Solution***

Let x1 be the number of hours allocated to work

Let x2 be the number of hours allocated to play

Max Z = x1 + 2x2

s.t.

$x1 + x2 \leq 10$

$x1 \geq x2 \Rightarrow x2 - x1 \leq 0$

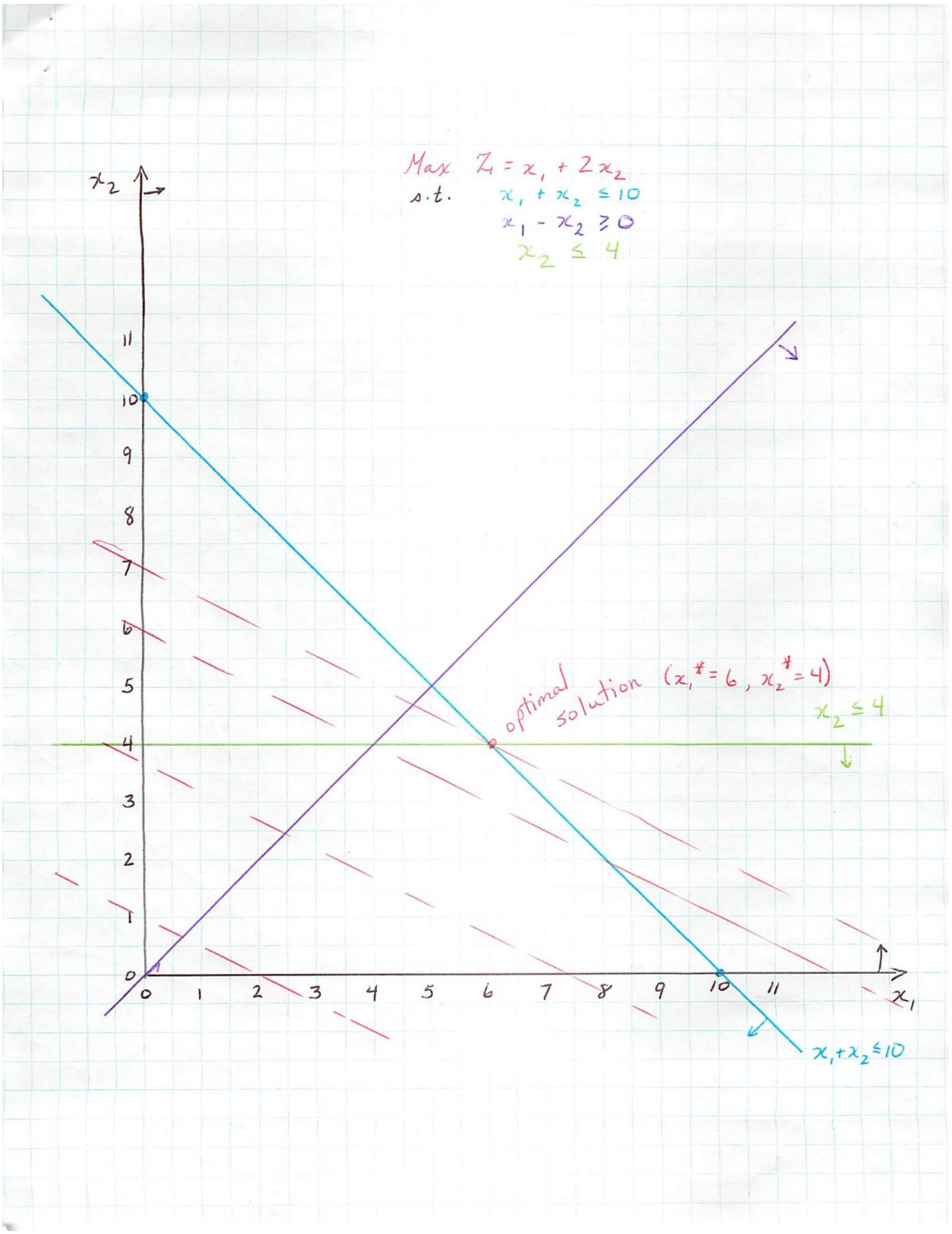
$x2 \leq 4$

$x1, x2 \geq 0$

*Simplex Solution*

See excel file

*Graphical Solution*


Max $Z = x_1 + 2x_2$
s.t. $x_1 + x_2 \leq 10$
$x_1 - x_2 \geq 0$
$x_2 \leq 4$
$x_2$
$x_1$
optimal solution $(x_1^* = 6, x_2^* = 4)$
$x_2 \leq 4$
$x_1 + x_2 \leq 10$
0 1 2 3 4 5 6 7 8 9 10 11

3. The Burroughs Garment Company manufactures men's shirts and women's blouses for Walmark Discount Stores. Walmark will accept all the production supplied by Burroughs. The production process includes cutting, sewing, and packaging. Burroughs employs 25 workers in the cutting department, 35 in the sewing department, and 5 in the packaging department. The factory works one 8-hour shift, 5 days a week. The following table gives the time requirements and profits per unit for the two garments:

| Minutes per unit | | | | |
|---|---|---|---|---|
| Garment | Cutting | Sewing | Packaging | Unit profit ($) |
| Shirts | 20 | 70 | 12 | 8 |
| Blouses | 60 | 60 | 4 | 12 |

Determine the optimal weekly production schedule for Burroughs.

***Solution***

Let x1 be the number of men's shirts produced

Let x2 be the number of women's blouses produced

Max Z = 8x1 + 12x2

s.t.

$20x1 + 60x2 \leq 25 * 8$ hours/day * 5 days/week * 60 minutes/hour

$70x1 + 60x2 \leq 35 * 8$ hours/day * 5 days/week * 60 minutes/hour

$12x1 + 4x2 \leq 5 * 8$ hours/day * 5 days/week * 60 minutes/hour

$x1, x2 \geq 0$

*Simplex Solution*

See excel file

*Graphical Solution*

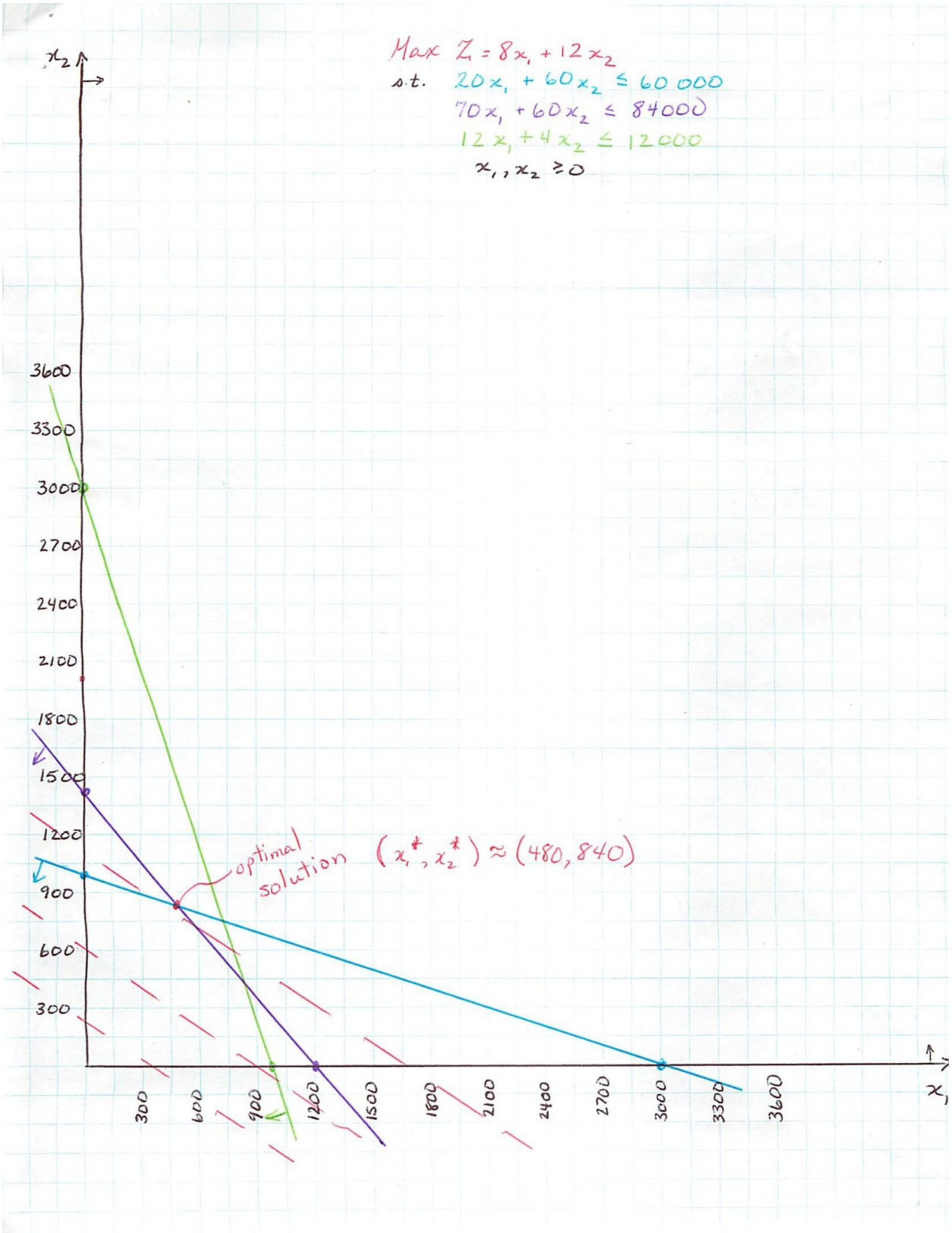
$$\text{Max } Z = 8x_1 + 12x_2$$

$$\text{s.t. } 20x_1 + 60x_2 \leq 60\,000$$

$$70x_1 + 60x_2 \leq 84000$$

$$12x_1 + 4x_2 \leq 12000$$

$$x_1, x_2 \geq 0$$



4. John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 5 and 12 hours a week, and in store 2, he is allowed between 6 and 10 hours. Both stores pay the same hourly wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on an ascending scale of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. How many hours should John work in each store?

***Solution***

Let x1 be the number of hours John works at store 1

Let x2 be the number of hours John works at store 2

Min Z = 8x1 + 6x2

s.t.

$x1 + x2 \geq 20$

$x1 \geq 5$

$x1 \leq 12$

$x2 \geq 6$

$x2 \leq 10$

$x1, x2 \geq 0$

***Solution***

*Simplex Solution*

See excel file

*Graphical Solution*


Min $Z_1 = 8x_1 + 6x_2$
s.t. $x_1 + x_2 \geq 20$
$x_1 \geq 5$
$x_1 \leq 12$
$x_2 \geq 6$
$x_2 \leq 10$
$x_1, x_2 \geq 0$
$x_2$
$x_1$
optimal solution
$(x_1^*, x_2^*) = (10, 10)$

5. Consider the following set of constraints:

$x_1 + 2x_2 + 2x_3 + 4x_4 \leq 40$

$2x_1 - x_2 + x_3 + 2x_4 \leq 8$

$4x_1 - 2x_2 + x_3 - x_4 \leq 10$

$x_1, x_2, x_3, x_4 \geq 0$

Solve the problem for each of the following objective functions.

(a) Maximize $Z = 2x_1 + x_2 - 3x_3 + 5x_4$

(c) Maximize $Z = 3x_1 - x_2 + 3x_3 + 4x_4$

*(Question 2, page 86 in text)*

***Solution***

See excel file