1- After one-pair, the next most common hands are two-pair and three-of-a-kind:

Three-of-a-kind: Three cards have one rank and the remaining two cards have two other ranks. e.g. {2♥,2♠,2♣,5♣,K♦}

- Calculate the probability of three-of-kind hand. **Ans: 0.0211**

One-pair: Two cards have one rank and the remaining thre cards have thre other ranks. e.g. {2♥,2♠,3♣,5♣,K♦}

- Calculate the probability of one-pair hand.

Solution: for a specific order (x,x,w,y,z):

Numerator: the first card could be any of 52 card, the second card should be the same as first card (3 choices), third card should be different than the 1st and 2nd cards (52-4=48 choices), forth card should be different than 1,2,and 3rd cards (52-4-4=44 choices), fifth card should be different than 1,2, 3rd and 4th cards (52-4-4-4=40 choices). We can reorder (x,x,w,y,z) in ${}_5C_2$ different ways.

Denominator: total number of possible selection: 52*51*50*49*48

P(one-pair-hand)= ${}_5C_2$ ×52*3*48*44*40/(52*51*50*49*48)=**0.422**

The data shown here represent the number of miles per gallon (mpg) that 30 selected four-wheel-drive sports utility vehicles obtained in city driving. Construct a frequency distribution, and analyze the distribution.

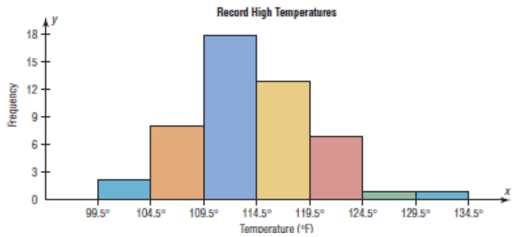
12	17	12	14	16	18
16	18	12	16	17	15
15	16	12	15	16	16
12	14	15	12	15	15
19	13	16	18	16	14

The completed ungrouped frequency distribution is

Class limits	Class boundaries	Frequency
12	11.5–12.5	6
13	12.5–13.5	1
14	13.5–14.5	3
15	14.5–15.5	6
16	15.5–16.5	8
17	16.5–17.5	2
18	17.5–18.5	3
19	18.5–19.5	1

Construct a histogram to represent the data shown for the record high temperatures

Class boundaries	Frequency
99.5–104.5	2
104.5–109.5	8
109.5–114.5	18
114.5–119.5	13
119.5–124.5	7
124.5–129.5	1
129.5–134.5	1



A student scored 65 on a calculus test that had a mean of 50 and a standard deviation of 10; she scored 30 on a history test with a mean of 25 and a standard deviation of 5. Compare her relative positions on the two tests.

Solution

First, find the z scores. For calculus the z score is

$$z = \frac{X - \overline{X}}{s} = \frac{65 - 50}{10} = 1.5$$

For history the z score is

$$z = \frac{30 - 25}{5} = 1.0$$

Since the z score for calculus is larger, her relative position in the calculus class is higher than her relative position in the history class.

Hospital records indicated that knee replacement patients stayed in the hospital for the number of days shown in the distribution.

Number of days stayed	Frequency
3	15
4	32
5	56
6	19
7	5
	127

Find these probabilities.

a. A patient stayed exactly 5 days.
b. A patient stayed less than 6 days.
c. A patient stayed at most 4 days.
d. A patient stayed at least 5 days.

Solution

a. $P(5) = \frac{56}{127}$

b. $P(\text{fewer than 6 days}) = \frac{15}{127} + \frac{32}{127} + \frac{56}{127} = \frac{103}{127}$
(Less than 6 days means 3, 4, or 5 days.)

c. $P(\text{at most 4 days}) = \frac{15}{127} + \frac{32}{127} = \frac{47}{127}$
(At most 4 days means 3 or 4 days.)

d. $P(\text{at least 5 days}) = \frac{56}{127} + \frac{19}{127} + \frac{5}{127} = \frac{80}{127}$
(At least 5 days means 5, 6, or 7 days.)

secondary schools were classified by the number of computers they had. Choose one of these schools at random.

Computers	1–10	11–20	21–50	51–100	100+
Schools	3170	4590	16,741	23,753	34,803

Choose one school at random. Find the probability that it has

a. 50 or fewer computers 0.295
b. More than 100 computers 0.419
c. No more than 20 computers 0.093

A game is played by drawing 4 cards from an ordinary deck and replacing each card after it is drawn. Find the probability that at least 1 ace is drawn.

Solution

It is much easier to find the probability that no aces are drawn (i.e., losing) and then subtract that value from 1 than to find the solution directly, because that would involve finding the probability of getting 1 ace, 2 aces, 3 aces, and 4 aces and then adding the results.

Let E = at least 1 ace is drawn and $\overline{E}$ = no aces drawn. Then

$$P(\overline{E}) = \frac{48}{52}\cdot\frac{48}{52}\cdot\frac{48}{52}\cdot\frac{48}{52}$$
$$= \frac{12}{13}\cdot\frac{12}{13}\cdot\frac{12}{13}\cdot\frac{12}{13} = \frac{20{,}736}{28{,}561}$$

Hence,

$$P(E) = 1 - P(\overline{E})$$
$$P(\text{winning}) = 1 - P(\text{losing}) = 1 - \frac{20{,}736}{28{,}561} = \frac{7825}{28{,}561} \approx 0.27$$

or a hand with at least 1 ace will occur about 27% of the time.

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Solution

Here, you must select 3 women from 7 women, which can be done in ${}_7C_3$, or 35, ways. Next, 2 men must be selected from 5 men, which can be done in ${}_5C_2$, or 10, ways. Finally, by the fundamental counting rule, the total number of different ways is $35 \cdot 10 = 350$, since you are choosing both men and women. Using the formula gives

$${}_7C_3 \cdot {}_5C_2 = \frac{7!}{(7-3)!3!}\cdot\frac{5!}{(5-2)!2!} = 350$$

A talk radio station has four telephone lines. If the host is unable to talk (i.e., during a commercial) or is talking to a person, the other callers are placed on hold. When all lines are in use, others who are trying to call in get a busy signal. The probability that 0, 1, 2, 3, or 4 people will get through is shown in the distribution. Find the variance and standard deviation for the distribution.

X	0	1	2	3	4
***P*(*X*)**	0.18	0.34	0.23	0.21	0.04

Should the station have considered getting more phone lines installed?

Solution

The mean is

$$\mu = \Sigma X \cdot P(X)$$
$$= 0\cdot(0.18) + 1\cdot(0.34) + 2\cdot(0.23) + 3\cdot(0.21) + 4\cdot(0.04)$$
$$= 1.6$$

The variance is

$$\sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$$
$$= [0^2\cdot(0.18) + 1^2\cdot(0.34) + 2^2\cdot(0.23) + 3^2\cdot(0.21) + 4^2\cdot(0.04)] - 1.6^2$$
$$= [0 + 0.34 + 0.92 + 1.89 + 0.64] - 2.56$$
$$= 3.79 - 2.56 = 1.23$$
$$= 1.2 \text{ (rounded)}$$

The standard deviation is $\sigma = \sqrt{\sigma^2}$, or $\sigma = \sqrt{1.2} = 1.1$.

No. The mean number of people calling at any one time is 1.6. Since the standard deviation is 1.1, most callers would be accommodated by having four phone lines because $\mu + 2\sigma$ would be $1.6 + 2(1.1) = 1.6 + 2.2 = 3.8$. Very few callers would get a busy signal since at least 75% of the callers would either get through or be put on hold.

A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

Solution

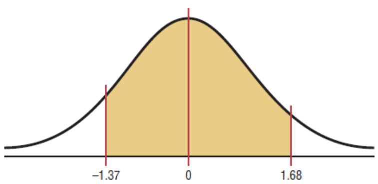
With the formulas for the binomial distribution and $n = 4$, $p = \frac{1}{2}$, and $q = \frac{1}{2}$, the results are

$$\mu = n \cdot p = 4 \cdot \tfrac{1}{2} = 2$$
$$\sigma^2 = n \cdot p \cdot q = 4 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} = 1$$
$$\sigma = \sqrt{1} = 1$$

Find the area between $z = +1.68$ and $z = -1.37$.

Solution

Step 1 Draw the figure as shown. The desired area is shown in Figure



Step 2 Since the area desired is between two given z values, look up the areas corresponding to the two z values and subtract the smaller area from the larger area. (Do not subtract the z values.) The area for $z = +1.68$ is 0.9535, and the area for $z = -1.37$ is 0.0853. The area between the two z values is $0.9535 - 0.0853 = 0.8682$ or 86.82%.

A survey found that women spend on average \$146.21 on beauty products during the summer months. Assume the standard deviation is \$29.44. Find the percentage of women who spend less than \$160.00. Assume the variable is normally distributed.

Solution

Step 1 Draw the figure and represent the area as shown in Figure

Step 2 Find the z value corresponding to \$160.00.

$$z = \frac{X - \mu}{\sigma} = \frac{\$160.00 - \$146.21}{\$29.44} = 0.47$$

Hence \$160.00 is 0.47 of a standard deviation above the mean of \$146.21, as shown in the z distribution in Figure

Step 3 Find the area, using Table E. The area under the curve to the left of $z = 0.47$ is 0.6808.

Therefore 0.6808, or 68.08%, of the women spend less than \$160.00 on beauty products during the summer months.

The average number of pounds of meat that a person consumes per year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

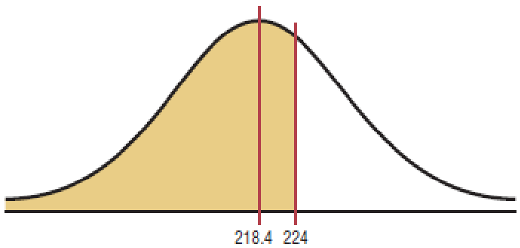
Source: Michael D. Shook and Robert L. Shook, *The Book of Odds*.

a. Find the probability that a person selected at random consumes less than 224 pounds per year.

b. If a sample of 40 individuals is selected, find the probability that the mean of the sample will be less than 224 pounds per year.

Solution

a. Since the question asks about an individual person, the formula $z = (X - \mu)/\sigma$ is used.



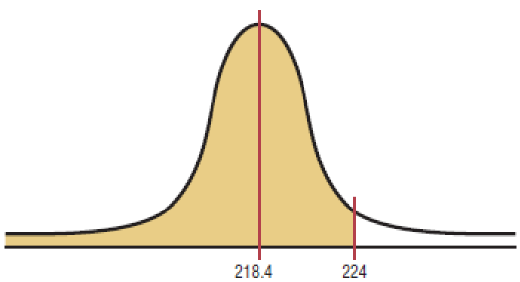
Distribution of individual data values for the population

The z value is

$$z = \frac{X - \mu}{\sigma} = \frac{224 - 218.4}{25} = 0.22$$

The area to the left of $z = 0.22$ is 0.5871. Hence, the probability of selecting an individual who consumes less than 224 pounds of meat per year is 0.5871, or 58.71% [i.e., $P(X < 224) = 0.5871$].

b. Since the question concerns the mean of a sample with a size of 40, the formula $z = (\overline{X} - \mu)/(\sigma/\sqrt{n})$ is used.



Distribution of means for all samples of size 40 taken from the population

The z value is

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{224 - 218.4}{25/\sqrt{40}} = 1.42$$

The area to the left of $z = 1.42$ is 0.9222.