Midterm Exam

Course Title:	Adv. Numerical Methods	**Course Code:**	ENGG*6090-S7
Date:	May 25, 2020	**Duration:**	2 hours

NOT TO BE REMOVED FROM THE EXAMINATION ROOM

Student Name:	
Student ID:	
Instructor:	Dr. Amin Komeili

INSTRUCTIONS TO STUDENTS

- Please write your name, ID#.
- Students are allowed to use the lecture notes.
- You are required to show a detailed solution. No marks will be given if the answer was not presented logically where all steps lead to the final solution.
- Print your answers clearly. If the marker cannot read the answer, it will be automatically assumed incorrect.
- The invigilators will not answer questions regarding the test paper. If you are unclear about a question on the test paper, state any assumptions that you make and then write your answer clearly.
- Your professionalism and consideration as ethical and responsible engineering students are required in this examination.

Question	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**Total**
Points:	15	10	35	25	15	100
Score:						

Question (1) **(15 marks)**

Solve the system of equations using the Gaussian Elimination method with partial pivoting.

$$6x + 4y + 13z = -23$$
$$2x + y - z = 4$$
$$-3x + 6y - z = 8$$

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Question (2) **(10 marks)**

Use the Bisection method to find an approximation to the root of $f(x) = e^{-x^2} + xtanx - 5$ in the range of [-5.5 , 2]. Continue the iteration until **one accurate significant digit** is guaranteed. Show your calculation for iteration 0 and 1.

Itr.	x_l	x_u	x_{mean}	f_l	f_u	f_{mean}	$\in \%$
0							
1							
2							
3							
4							
5							
6							
8							
9							
10							
11							

Question (3) Solve the following systems of equation with start point $(x_0, y_0, z_0) = (1,1,0)$:

$$y - cos(x) = 0$$

$$y = \sqrt{x}$$

$$x + y - z = 0$$

(a) Using the Fixed point iteration up to 4 iterations. (Complete the table.) **(10 Mark)**

$g_1(x, y, z) =$				
$g_2(x, y, z) =$				
$g_3(x, y, z) =$				
$Iteraion\ (i)$	x_i	y_i	z_i	$\lvert \in_x \rvert \%$
0	1	1	0	-
1				
2				
3				
4				

(b) If your solution did not converge, what could be the reasons? Support your answer with calculations. **(10 Marks)**

(c) Using the Newton Raphson method. Continue the iteration until error for x is less than **5%.** (Complete the table.) **(15 Marks)**

Table 1-detailed solution for iteration 1

	$[Jacobian]\{\Delta x_i\} = [b_i(x, y, z)]$
$Jacobian(x, y, z) =$	
$Jacobian_0(1,1,0) =$	
$[b_0(1,1,0)] =$	
$\begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} =$	

$Iteraion\ (i)$	x_i	y_i	z_i	$\in_x\%$
0	1	1	0	-
1				
2				
3				
4				

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Question (4) The following table lists values of the sin(X) at various points.

x	0.0	0.1	0.3	0.7	0.8	0.9	1.3	1.9	2.2
Sin(x)	0.0	0.099833	0.29552	0.644218	0.717356	0.783327	0.963558	0.94630	0.808496

(a) determine the interpolation function at x=0.85, using a **third-order** polynomial function from the Newton divided-difference method. **(20 Marks)**

(b) Calculate the $sin(0.85)$ using the interpolation function you obtained in part (a) and determine the **relative true error**. **(5 Marks)**

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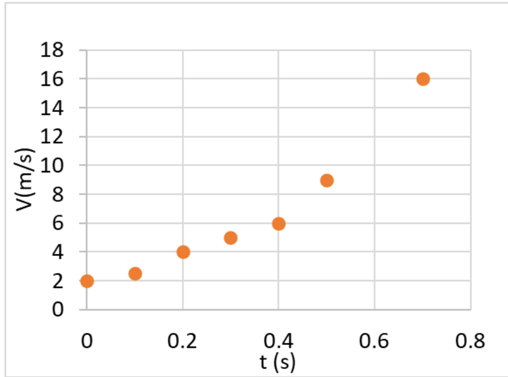
Question (5) The velocity of a car is measured at different times. An exponential regression curve is going to be used to estimate the velocity of the car.

t (s)	V (m/s)
0	2
0.1	2.5
0.2	4
0.3	5
0.4	6
0.5	9
0.7	16



(a) Transfer the data to the logarithm basis and determine the regression line $y^* = at + b$. **(13 marks)**

	t	V						
	0	2						
	0.1	2.5						
	0.2	4						
	0.3	5						
	0.4	6						
	0.5	9						
	0.7	16						
$\sum$								

(b) Estimate the velocity of the car at t=1s. **(2 marks)**

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